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Mining Motion Data of Scoliotic Spine in the Coronal Plane to Predict the Spine in Lateral Bending Positions

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Abstract. Reducing the number of X-rays required for scoliosis surgery planning is essential for mitigation of radiation risks. In this regard, we aimed to study prediction of scoliotic spine curvatures in lateral bending positions by using motion data acquired from sequence of the spine movement from the erect to bending positions. We utilized our patient-specific multi-body kinematic model that is a chain of micro-scale motion segments (MMSs) to reconstruct the curvatures and acquire the spine motion data. Hough transform was adopted to analyze the motion data. It was found that there is an excellent linear relationship ($R^2 = 0.93 \pm 0.09$) between the motion data of MMSs placed between each two successive vertebrae. Using the linear relationships, we could make good estimates of the spine curvatures (Root-mean-square-error (RMSE) = 0.0207 mm) and the key parameters for scoliosis surgery planning; RMSEs of curvature angle, spinal mobility, and spinal flexibility were 0.0123°, 0.0089°, and 0.0002 respectively. This study showed that scoliotic spine curvatures in the bending positions and the key parameters for surgery planning can be predicted by using X-ray of the erect spine. Such an important insight can lead to reduction of the number of X-rays in scoliosis standard care to mitigate the radiation risks, which is one of the surgeons' main priorities.

Keywords*:* Erect Spine, Lateral Bending Positions, Multi-body Kinematic Model, Prediction, Scoliosis, Spine Curvature, Spine Motion Data.

1 Introduction

Scoliosis is a complex 3-dimensional structural deformity of the human spine. The severe deformities often require surgery for the correction [1]. Before the surgery, several X-rays are taken from the scoliotic spine so as to measure the key parameters (e.g. curvature angle, spinal mobility, and spinal flexibility) required for planning the surgery (e.g. for selecting fusion levels). The X-rays include the erect and lateral bending positions in the coronal plane [2], [3]. X-ray imaging exposes the patients to harmful radiation. To mitigate the radiation risk, reducing the number of the required X-rays can be potentially helpful.

As an alternative to the X-ray imaging, surface measurement methods, such as motion capture [4], [5], inclinometer [6], and kyphometer [7] methods, have been adopted to measure the key parameters such as the spinal mobility and spinal flexibility. However, Hresko et al. [8] showed that there is no significant statistical correlation between the measurements by using the surface measurement methods and the X-rays (which are the gold standard for the measurements [9-12]). Another alternative can be prediction of the scoliotic spine curvatures in the positions for measuring the key parameters. To the best of our knowledge, little mention has been made with regards to study on such a prediction [13].

We aim to study prediction of the lateral bending positions in the coronal plane from the erect spine. This paper determines whether there is a mathematical relationship between the spine curvatures in the erect and lateral bending positions, and determines whether the relationship can be used for the prediction of the spine curvatures and the abovementioned key parameters. To accomplish the aim, we analyze the spine motion data in the coronal plane. The motion data is acquired from X-rays of the spine in the erect and lateral bending positions, considering these X-rays as the representative of a sequence of spine movement from the left bending to erect to right bending positions. This study focuses on adolescent idiopathic scoliosis (AIS), the most common form of the scoliosis (≈ 80% of all cases [14]). The AIS patients are predominantly female [15].

2 Scoliotic Spine

This section briefly explains the scoliotic spine, its evaluation, and the key parameters for planning the scoliosis surgery. Scoliosis is a side-to-side curvature of the spine with a twisting of the vertebral column about its axis (Fig. 1a) [16], [17]. The scoliotic deformity affects the lumbar and thoracic regions of the spine from the fifth vertebra (L5) of the lumbar region to the first vertebra (T1) of the thoracic region, and causes S- and/or C-shape spine curvature in the coronal plane [17]. It has been diagnosed in 1.5% to 3% of the population [1].

The gold standard for evaluating the scoliotic spine is curvature angles measured on two-dimensional radiographs [1]. In the coronal plane, there are several methods to measure the angle, such as Analytic Cobb method [18], [19], Cobb method [20], or Ferguson method [21]. Analytic Cobb method measures the curvature angle between the perpendiculars at inflectional points of the curvature (term number 48 in [22]) (Fig. 1b).

In addition to the curvature angle, the spinal mobility, and spinal flexibility are the key parameters for the scoliosis surgery planning [13], [2], [3]. Spinal mobility is the angle between the lines connecting the mid-points of the lowest and uppermost vertebrae in the erect and left/right bending positions (Fig. 2). It can be defined for a part of the spine, e.g. thoracic region or the whole spine. Spinal flexibility is the difference of spinal movement (excursion) from the erect to lateral bending positions [23]. The scoliosis flexibility index is calculated by Equation 1 according to [8].

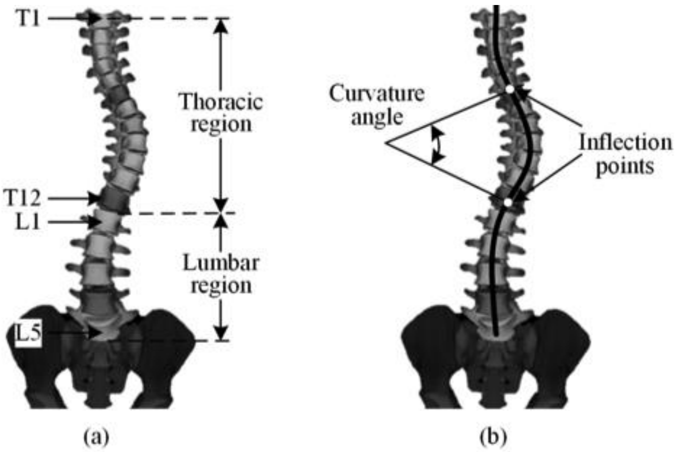

Fig. 1. (a) The scoliotic spine and the spine regions, and (b) the curvature angle.

$$\text{Flexibility index} = \frac{\left(\text{Curvature angle}_{\text{erect}} - \text{Curvature angle}_{\text{bending}}\right)}{\text{Curvature angle}_{\text{erect}}} \tag{1}$$

3 Methods

To identify mathematical relationships between the spine curvatures, the curvatures are divided into small segments, and the motion data of the segments are studied. To do this, we utilize our patient-specific kinematic model of the scoliotic spine [24] developed for reconstruction of the spine curvatures and representation of the spine movement in the coronal plane. The model is a chain of micro-scale motion segments (MMSs) laying on the spine curvatures. The model gives the orientations (θ) of MMSs with respect to their inferior MMS in the spine movement. Hough transform [25] is adopted to study the relationships between θ of MMSs in the erect and left/right bending positions. Hough transform is a feature extraction technique from image processing and computer vision. It helps finding shapes by a voting procedure; in our case, the shapes are the relationships between θ of MMSs.

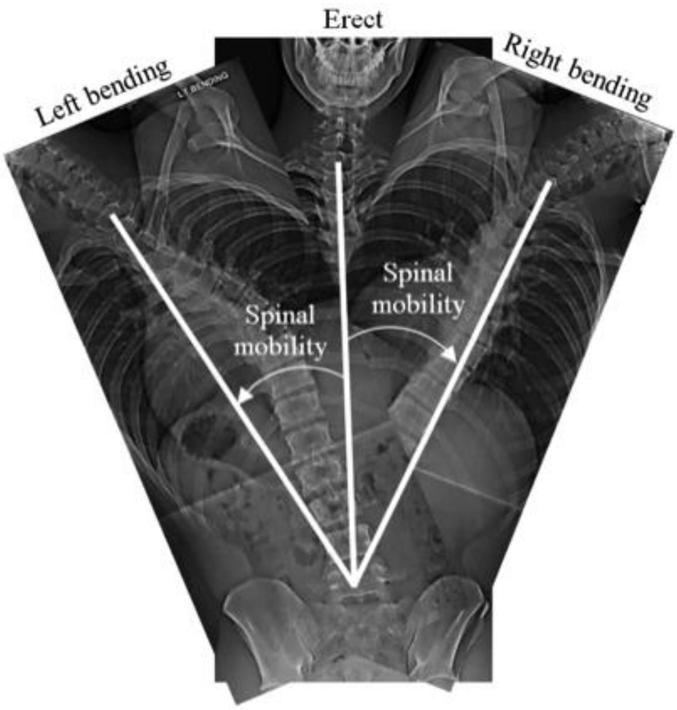

Fig. 2. Spinal mobility in bending towards left and right.

3.1 Subjects

The current study has been approved by the domain specific review board (DSRB) and ethics committee. All patients involved in this study had been properly consulted, and their approval and informed consents were obtained. Following the DSRB approval and obtaining the proper informed consents, pre-operative X-rays of 18 patients with AIS were used for the study (Table 1). The X-rays were taken in three posterior-anterior positions in the coronal plane; erect, left bending, and right bending positions. The patients had no neurological deterioration, and they were admitted to hospital for surgical treatment. There were 12 females and 6 males between the ages of 12 and 19 years (mean age of 15 years). Cobb angle of the main curves (the largest Cobb angle) ranged from 46° to 86°, and the average and median Cobb angles were 56° and 53° respectively.

Table 1. Descriptive data of the patients.

Patient	Gender	Age (year)	Lenke classi-fication	Cobb angle (º) of the main curve
1	Female	13	1A	49
2	Female	15	1B	53
3	Female	16	1C	46
4	Female	12	1C	48
5	Female	13	2A	53
6	Female	16	2B	53
7	Female	19	2C	48
8	Female	14	2C	55
9	Female	15	3A	59
10	Female	13	4C	86
11	Female	12	5C	62
12	Female	14	6C	59
13	Male	14	2A	59
14	Male	19	2B	48
15	Male	18	2B	61
16	Male	18	3B	46
17	Male	14	3C	53
18	Male	19	3C	70

3.2 Data Acquisition for Reconstruction of the Spine Curvatures

The three X-rays were analyzed to measure the location (LOC) and orientation (Θ) of the vertebrae in order to obtain the spine curvatures. LOC was defined as the location of the mid-point of the vertebral body in the coronal plane. The mid-point (Fig. 3a) is the intersection of the line drawn from the upper left corner to the lower right of the vertebral body and the line drawn from the upper right to the lower left of the vertebral body [26]. Θ was considered as the orientation of the line (Fig. 3a) passing through the center of the upper and lower endplates of the vertebra (in accordance with the definition of ‘vertebral lateral rotation’ [27]). LOC and Θ were measured for the vertebrae from L4 to T2 because the X-rays at L5 and T1 were often suboptimal for the measurement.

LOC and *Θ* were defined in the global coordinate system (*G*) represented by XYZ on the lowest vertebra (L4) in each X-ray (Fig. 3b). *G* has its origin at the mid-point of L4 [24], [28]. Its X-axis and Y-axis define the anterior and left directions respectively according to Three-dimensional Terminology of Spinal Deformity provided by Scoliosis Research Society [29]. Z-axis is parallel to the line that shows the orientation of L4. The plane YZ is the coronal plane. By referring to the definition of *G*, LOC and *Θ* of L4 are (0,0) and zero respectively. Next section explains derivation of the spine curvatures by using LOC and *Θ*.

The measurements were done manually by two experts in radiographic measurements of the spine, three times. Then, the mean values of the measurements were considered. All the measurements were supervised by G. Liu (one of the authors) who is an experienced scoliosis surgeon at National University Hospital, Singapore. The intra- and inter-observer reliabilities of the measurements were evaluated by using Pearson correlation analysis. The intra-observer reliabilities of the measurements were 0.95 ± 0.04 for expert-1 and 0.93 ± 0.05 for expert-2. The inter-observer coefficient was 0.91. These agreements are excellent according to Richards et al. [30] and can show the repeatability and reliability of the measurements.

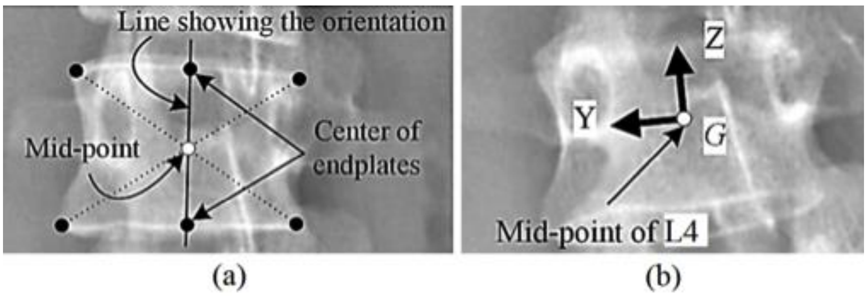

Fig. 3. (a) Location and orientation of the vertebrae, and (b) the global coordinate system.

3.3 Reconstruction of the Spine Curvature

The scoliotic spine curvature in the coronal plane is a curved line that passes through the mid-points of all the vertebrae (Fig. 3a) according to Stokes [27]. Polynomials are used to define the curvatures [24], [31], [32] and fitted to LOC and *Θ* by using the linear least squares method (Appendix A); a detailed description is provided in our previous work [24]. The order of each polynomial is adjusted so as to find the best fitting, i.e. least root-mean-square-errors (RMSE) of LOC and *Θ*. RMSEs of LOC and *Θ* were 0.18 ± 0.06 mm and $0.16 \pm 0.06^{\circ}$ respectively. The small RMSEs show that the fitted polynomials can give good estimates of the locations and orientations of the vertebrae.

The chain of MMSs lays on the polynomial to reconstruct the spine curvature. An MMS consists of a rigid link and a 1-DOF rotary joint (Fig. 4a). The links are equal in length. The chain (Fig. 4b) is constrained at the first MMS attached to the mid-point of L4, and cannot translate with respect to this mid-point. The last MMS corresponds to the mid-point of the uppermost vertebra T2. The chain lays on the spine curvature in each position (Fig. 4b) and estimates the curvature, and the location and orientation of the vertebrae in the position. To reconstruct the spine curvature in a position, θ (the angle of the rotary joints) of MMSs must be identified. θ of an MMS is defined with respect to x-axis attached to its inferior MMS according to Denavit-Hartenberg convention [33]. For example, θ_1 is the angle between x_1 and x_0, and it is measured with respect to x_0 in the counterclockwise direction (Fig. 4b).

The chain of MMSs is fully characterized to a given patient by specification of the number of MMSs and the length of their links. In this study, the total number of MMSs is considered 1000 according to our previous works [13], [24], [31], and the length of the links is 0.37 ± 0.03 mm.

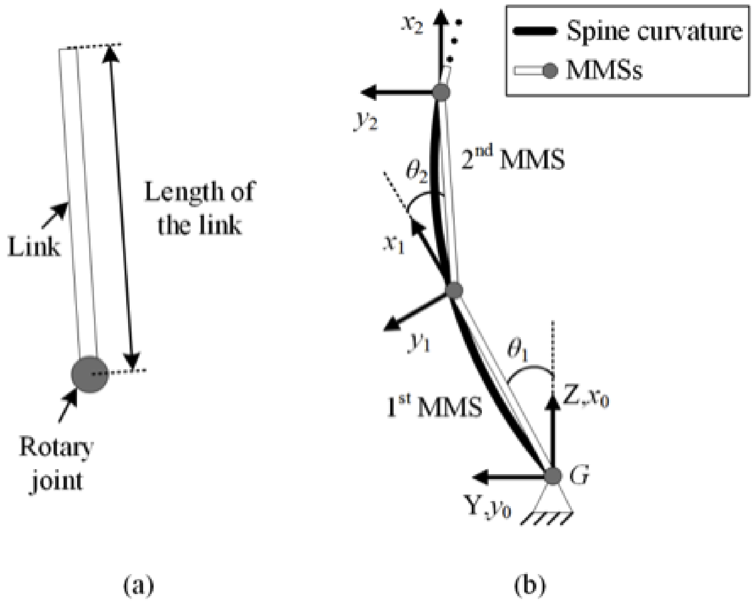

Fig. 4. The kinematic model, (a) an MMS and (b) the chain of MMSs that lays on the spine curvature and the coordinate systems

3.4 Mining of the Spine Motion Data

For each patient, θs of MMSs in the erect and lateral bending positions are provided by the kinematic model. Then, we utilize Hough transform for clustering the ordered pairs of (θ_{erect},$\theta_{left/right}$). In other words, we try to find MMSs whose $\theta_{left/right}$ are related to their θ_{erect} in the similar way, i.e. their rotations draw a certain shape in the $\theta_{erect}\theta_{left/right}$ plane.

Hough transform helps finding shapes by a voting procedure. For example, to extract straight lines (i.e. $\theta_{left/right}= \alpha\cdot\theta_{erect}+\beta$), θ_{erect} and $\theta_{left/right}$ are considered as the slopes and intercepts of straight lines in the plane of $\alpha\beta$ respectively, i.e. $\beta=-\theta_{erect}\cdot\alpha+\theta_{left/right}$. The intersections of the lines in the plane of $\alpha\beta$ are counted, and then, (α,β) with high frequency within their nearby are found. These (α,β) give the number of the straight lines and their slopes and intercepts in the plane of $\theta_{erect}\theta_{left/right}$, respectively.

4 Results

4.1 Hough Transform Analysis

For each spine, 14.1 ± 0.4 clusters of collinear pairs of (θ_{erect},$\theta_{left/right}$) were found by using the Hough transform (Fig. 5a). Examples of two clusters in the $\alpha\beta$ plane for a patient are shown in Fig. 5b. The vertical axis gives the number of times that the pairs of (θ_{erect},$\theta_{left/right}$) give a pair of (α,β). It was found that about 93% of the pairs in a cluster belonged to MMSs placed between the mid-points of two successive vertebrae. Thus, the averagely 14 clusters for each patient can be corresponded to the 14 intervertebral discs in each spine curvature. This can be attributed to the fact that MMSs between the mid-points of two successive vertebrae possess similar mechanical properties as they represent a single intervertebral disc.

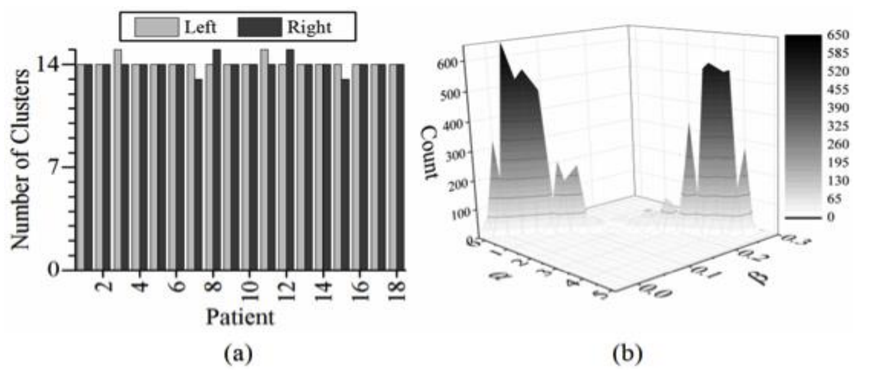

Fig. 5. The number of the identified clusters for each patient in the left and right bending positions, and (b) two clusters in the $\alpha\beta$ plane for a patient.

4.2 The Relationships between the Spine Curvatures

The results of the Hough transform analysis suggest that there can be a linear relationship between θ_{erect} and $\theta_{left/right}$ of MMSs placed between the mid-points of two successive vertebrae, i.e. $\theta_{left/right} \approx \alpha \cdot \theta_{erect} + \beta$. In total, 504 linear relationships (14 vertebrae × 2 bending positions × 18 patients) were identified. The coefficient of determination (R^2) for the linear relationships was 0.93 ± 0.09. The confidence interval of the mean of R^2 was between 0.92 and 0.95, with the confidence level of 95%. According to [30], [34], the high R^2 can show that there can be an excellent linear relationship between θ_{erect} and $\theta_{left/right}$ of MMSs between each two successive vertebrae.

The curvatures of 36 spines (1 curvature × 2 bending positions × 18 patients) were reconstructed by using the estimated $\theta_{left/right}$, and compared with the curvatures reconstructed by the measured $\theta_{left/right}$. RMSE of the curvatures was 0.0207 mm (Fig. 6a). The small RMSE can demonstrate that the scoliotic spine curvature in the left and right bending positions can be predicted by using the identified linear relationships.

There is one more step to demonstrate that the linear relationships can help to predict the key parameters for the surgery planning because to reduce the number of X-rays, the key parameters must be accurately predicted on the spine curvatures reconstructed by using the estimated θ. Therefore, the parameters measured on the estimated curvatures were compared with the actual ones measured on the X-ray. We adopted Analytic Cobb method to measure the curvature angle since the chain of MMSs can estimate the curvature. RMSE of the curvature angle was 0.0123º (Fig. 6b). RMSEs of the spinal mobility (0.0089, Fig. 6c) and spinal flexibility (0.0002, Fig. 6d) in the lateral bending positions were also small, showing that the parameters measured by using the estimated curvatures are good estimates of the parameters measured by using the actual curvatures. Overall, we showed that the scoliotic spine curvature and the key parameters for the scoliosis surgery planning can be accurately predicted by using the identified linear relationships and θ of MMSs measured only in the erect position.

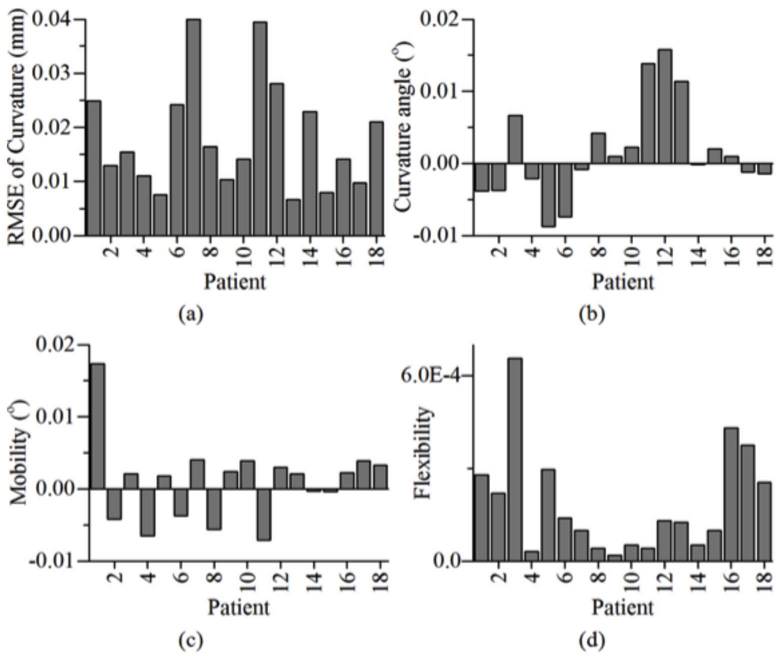

Fig. 6. The errors of (a) the curvature, (b) curvature angle, (c) mobility, and (d) flexibility index

5 Discussion

This study showed that the scoliotic spine curvatures in the lateral bending positions can be predicted from geometrical information of the spine curvature in the erect position. Through mining the spine motion data, it was demonstrated that by defining linear relationships between the angles of the rotary joints of MMSs in the erect and bending positions, we can predict the spine curvatures and the key parameters for scoliosis surgery planning. To acquire the motion data and study the prediction, we utilized our patient-specific kinematic model that reconstructs the spine curvature by a chain of micro-scale motion segments. This allowed us to perform a deep analysis in the relationships between the spine curvatures in the erect and bending positions because the chain provided the motion data of the small parts of the spine (i.e. the orientations of MMSs from the erect to lateral bending positions).

The prediction errors depend on the estimation errors of the slopes (α) and intercepts (β) of the linear relationships. Thus, the sensitivity of errors to the estimation errors of α and β is one of the important factors that can assess the capability of the proposed approach to the prediction. To determine the sensitivity, we introduced the errors of -20% to 20% (in steps of 2%) into α and β in this study. RMSEs of the spine curvature (E_C) and the curvature angle (E_{ANG}) were obtained in the presence of the introduced errors (Fig. 7).

For the considered range of the estimation errors, the worst E_C and E_{ANG} were 1.73 mm and 2.45º respectively. The worst cases occurred when the estimation errors of α and β were (20%, -20%) or (-20%, 20%). The small E_C and E_{ANG} (Fig. 7) imply that the spine curvature and the curvature angle can be reasonably predicted in the presence of the large (± 20%) estimation errors of α and β. In addition, the maximum RMSEs in the range of estimation errors of Δ (i.e. $-\Delta \leq \alpha \leq \Delta$ and $-\Delta \leq \beta \leq \Delta$) can be related to Δ for the prediction errors, where $\Delta \geq 0$. The relations are: $E_C \leq 0.086 \cdot \Delta$ mm (Fig. 7a) and $E_{\mathrm{ANG}} \leq 0.122 \cdot \Delta^{o}$ (Fig. 7b). The small coefficients of Δ for the relations can show the low sensitivity of the prediction errors to the estimation errors of α and β. These relations can also help to set the estimation errors so as to satisfy certain prediction requirements. For example, for the prediction requirement '$E_C \leq 0.5$ mm', Δ is $\leq$ 5.81%. Overall, it can be concluded that the RMSEs may not be significantly affected by the estimation errors of α and β. This can reaffirm the feasibility and capability of our approach to the prediction.

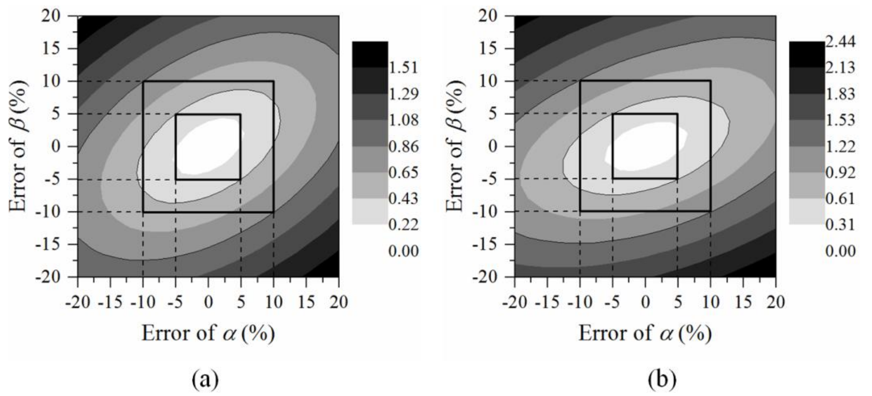

Fig. 7. RMSEs against the estimation errors of α and β, (a) E_C and (b) E_{ANG}.

In addition to the spine curvature, further studies should be done on the estimation of the spine shape including the locations and orientations of the vertebrae to evaluate the capability of the proposed method for the prediction of the spine in the bending positions. As a future work, we are evaluating our method for estimation of the spine shape.

In this study, the erect position was considered as the reference position to characterize the chain and to study the prediction because it is the reference position for evaluating the scoliotic spine, its surgery planning, and assessment of the spinal mobility and flexibility [20]. In addition, X-ray of the erect position is part of scoliosis routine standard care [8] and cannot be removed from the scoliosis standard care. Therefore, the erect position is an excellent choice to provide information for predicting the spine curvature in the lateral bending positions [13].

The majority of the patients in this study had curve types of 1, 2, and 3 according to Lenke classification [2], considering that these types affect around 75% of the population of the scoliotic patients [35]. Involving more samples of the other curve types can help to better generalize the results.

6 Conclusions

In this paper, we studied prediction of the scoliotic spine curvatures in the lateral bending positions by using spine motion data from the erect to the bending positions. This paper is the first to study such prediction, and as a result, it has provided an important insight: there can be linear relationships between the scoliotic spine curvatures in the erect and lateral bending positions, and the relationships can be used to predict the spine curvatures in the bending positions and the key parameters for planning the scoliosis surgery, i.e. curvature angle, spinal mobility, and spinal flexibility. This study is an important step towards developing a method for the prediction of the lateral bending positions. By such a method, we can obviate the need for the X-rays of the scoliotic spine in the lateral bending positions in planning the surgery in order to mitigate the radiation risks.

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Appendix A

The polynomial (f) of a position is given by Equation A1 in the global coordinate system. f has no first-degree and constant terms, since as mentioned in section Data acquisition, the location and orientation of LV are zero, i.e. $f(0)=0$ and $f'(0)=0$.

$$f(\mathrm{Z}) = \delta_t \mathrm{Z}^t + \delta_{t-1}\mathrm{Z}^{t-1} + \cdots + \delta_2 \mathrm{Z}^2 \tag{A1}$$

Where, δ_t, …, δ_2 are the coefficients. t is the order of the polynomial.

Equation A2 expresses the coefficients of f by using the linear least squares method.

$$\boldsymbol{\delta} = (\mathbf{Z}^{\mathrm{T}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathrm{T}}\mathbf{Y} \tag{A2}$$

Where, $^{\mathrm{T}}$ stands for transpose operation. Equation A3 gives $\boldsymbol{\delta}$ (the vectors of the coefficients), $\mathbf{Y}$ (the measured parameters), and $\mathbf{Z}$ (the design matrix). The upper and lower halves of $\mathbf{Y}$ and $\mathbf{Z}$ correspond to the measured locations and orientations respectively.

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_t \\ \delta_{t-1} \\ \vdots \\ \delta_2 \end{bmatrix} \qquad \mathbf{y}_i = \begin{bmatrix} \mathrm{Y}_1 \\ \mathrm{Y}_2 \\ \vdots \\ \mathrm{Y}_k \\ \tan \Theta_1 \\ \tan \Theta_2 \\ \vdots \\ \tan \Theta_k \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{The Upper Half} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{The Lower Half} \end{matrix}$$

$$\mathbf{X}_i = \begin{bmatrix} \mathrm{Z}_1^t & \mathrm{Z}_1^{t-1} & \cdots & \mathrm{Z}_1^2 \\ \mathrm{Z}_2^t & \mathrm{Z}_2^{t-1} & \cdots & \mathrm{Z}_2^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Z}_k^t & \mathrm{Z}_k^{t-1} & \cdots & \mathrm{Z}_k^2 \\ t\mathrm{Z}_1^{t-1} & (t-1)\mathrm{Z}_1^{t-2} & \cdots & 2\mathrm{Z}_1 \\ t\mathrm{Z}_2^{t-1} & (t-1)\mathrm{Z}_2^{t-2} & \cdots & 2\mathrm{Z}_2 \\ \vdots & \vdots & \ddots & \vdots \\ t\mathrm{Z}_k^{t-1} & (t-1)\mathrm{Z}_k^{t-2} & \cdots & 2\mathrm{Z}_k \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{The Upper Half} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{The Lower Half} \end{matrix} \tag{A3}$$

Where, the indices 1 to *k* correspond to the lowest to uppermost vertebrae.